FINANCIAL RISK MANAGEMENT · MARKET RISK

Option Greeks

Introduction to the Basics — Delta, Gamma & Vega and building Neutral Portfolios

A step-by-step handout with solved examples, Excel formulas (NORM.S.DIST, MINVERSE, MMULT) and a companion workbook where every number is a live formula.

Δ	Γ	ν	Θ	ρ
DELTA	**GAMMA**	**VEGA**	**THETA**	**RHO**
∂V/∂S	∂²V/∂S²	∂V/∂σ	∂V/∂t	∂V/∂r
Price risk (1st order)	Curvature risk (2nd order)	Volatility risk	Time decay	Interest-rate risk

What you will be able to do after this handout

Before you begin

Start with **Section 0 — Pre-requisites** (option payoffs, moneyness, BSM basics) if any of these feel rusty.

1	Define **Delta**, compute it as **$N(d_1)$** for a call and **$N(d_1) - 1$** for a put, and read a **position delta**.
2	Build a **delta-neutral** portfolio and see — in three illustrations — why it must be **rebalanced**.
3	Explain **Gamma** as the **second-order derivative**, its sign for long/short positions, and build a **Gamma + Delta neutral** book.
4	Calculate **Vega**, and solve a 2×2 system (by hand and with Excel **MINVERSE / MMULT**) for a **Gamma + Vega + Delta neutral** book.
5	Explain why desks do **not** run Theta-neutral or Rho-neutral books.
6	Solve three short **caselets** on delta, gamma-delta and gamma-vega-delta hedging.

The common data used in every example

Item	Value	Note
Underlying price S_0	₹100	Spot price today
Risk-free rate r	5% p.a. (cont. comp.)	Black-Scholes-Merton input
Volatility σ	20% p.a.	Implied volatility
Option A (we hold / write)	Call, K = 100, T = 0.50 yr	Price ₹6.8887
Option B (hedge option 1)	Call, K = 105, T = 0.25 yr	Price ₹2.4779
Option C (hedge option 2)	Call, K = 95, T = 1.00 yr	Price ₹13.3465
Position	Short 1,000 calls of A	Each option is on 1 share (multiply by lot size in practice)

0 Pre-requisites to master this handout

Option payoffs · Moneyness · Black-Scholes-Merton basics · Maths toolkit

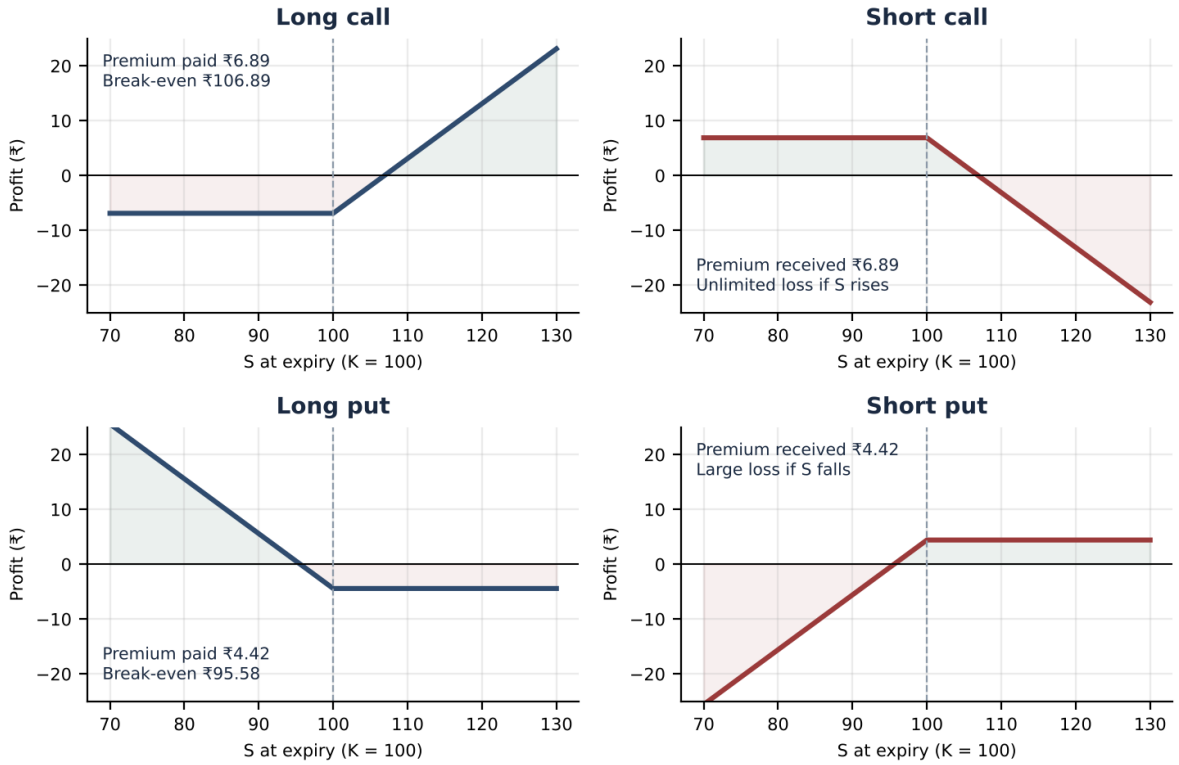
P.1 Readiness checklist

Topic	What you should be comfortable with	Used in
Option payoffs	Call/put, long/short, payoff vs profit, premium, break-even	Sign of Δ, Γ, position Greeks
Moneyness	ITM / ATM / OTM, intrinsic value, time value	Why Δ, Γ, Vega change with S
BSM model	Inputs, assumptions, d_1, d_2, N(·), call & put prices	All Greek formulas
Normal distribution	N(x) = cumulative probability, N′(x) = density	Delta, Gamma, Vega
Calculus idea	Derivative = slope, 2nd derivative = curvature	Delta (1st), Gamma (2nd)
Linear equations	Solve 2 equations in 2 unknowns; matrix inverse	Gamma + Vega neutral book
Excel	NORM.S.DIST, LN, EXP, SQRT, MINVERSE, MMULT, MDETERM	Companion workbook

P.2 Option payoffs — the starting point

A **call** gives the buyer the right (not the obligation) to **buy** the underlying at the strike K; a **put** gives the right to **sell** at K. The buyer (**long**) pays a premium; the writer (**short**) receives it and takes the obligation. A long and a short position are exact mirror images (zero-sum).

Position	Payoff at expiry	Profit = Payoff − Premium	Max gain	Max loss
Long call	$\max(S_T - K, 0)$	$\max(S_T - K, 0) - c$	Unlimited	Premium c
Short call	$-\max(S_T - K, 0)$	$c - \max(S_T - K, 0)$	Premium c	Unlimited
Long put	$\max(K - S_T, 0)$	$\max(K - S_T, 0) - p$	K − p	Premium p
Short put	$-\max(K - S_T, 0)$	$p - \max(K - S_T, 0)$	Premium p	K − p



Link to the Greeks

Long call and short put **gain when S rises** → positive delta. Short call and long put **gain when S falls** → negative delta. Every long option has a **kinked (convex)** payoff → positive gamma; every short option is **concave** → negative gamma. Keep these four pictures in mind throughout.

P.3 Moneyness — ITM, ATM, OTM

Moneyness	Call	Put	Intrinsic value	Typical call Δ
In-the-money (ITM)	S > K	S < K	Positive	Close to +1
At-the-money (ATM)	S = K	S = K	Zero	About +0.5
Out-of-the-money (OTM)	S < K	S > K	Zero	Close to 0

Option price = Intrinsic value + Time value

Intrinsic: call max(S − K, 0), put max(K − S, 0)

S (K = 100, T = 0.5)	Call price	Call intrinsic	Call time value	Put price	Put intrinsic	Put time value	Call Δ
₹90 (OTM call)	2.3494	0.00	2.3494	9.8804	10.00	-0.1196	0.3094
₹100 (ATM)	6.8887	0.00	6.8887	4.4197	0.00	4.4197	0.5977
₹110 (ITM call)	14.0754	10.00	4.0754	1.6064	0.00	1.6064	0.8216

Observe

Time value is largest **at-the-money** — the same place Gamma and Vega peak. Note the deep-ITM **European put** at S = 90 has a slightly **negative** time value (−0.12): it cannot be exercised early, and the strike is received only at expiry (discounted). An American put could never trade below intrinsic.

P.4 Black-Scholes-Merton (BSM) model basics

$$c = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \quad p = K \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$
$$d_1 = [\ln(S/K) + (r + \sigma^2/2)T] / (\sigma\sqrt{T}) \quad d_2 = d_1 - \sigma\sqrt{T}$$

Key assumptions

1	Share price follows geometric Brownian motion → prices are **lognormal**, returns normal.
2	Volatility σ and risk-free rate r are **constant** and known over the option's life.
3	**European** exercise; no dividends during the option's life (extensions exist for both).
4	Frictionless markets: no transaction costs or taxes, continuous trading, short selling allowed, no arbitrage.
5	Risk-neutral valuation: $N(d_2)$ = risk-neutral probability that a call finishes in-the-money; $N(d_1)$ = the call's delta.

Input ↑ (others fixed)	Call price	Put price	Greek that measures it
Underlying price S	↑	↓	Delta (and Gamma)
Strike K	↓	↑	—
Time to expiry T	↑	↑ (usually)	Theta (with sign reversed)
Volatility σ	↑	↑	Vega
Risk-free rate r	↑	↓	Rho

Maths toolkit in one box

A Greek is simply the **partial derivative** of the BSM price with respect to one input. Delta = $\partial V/\partial S$ is the slope; Gamma = $\partial^2 V/\partial S^2$ is the curvature. N(x) in Excel is `NORM.S.DIST(x,TRUE)` and the density N′(x) is `NORM.S.DIST(x,FALSE)`. Two equations in two unknowns are solved with `MMULT(MINVERSE(M),b)`.

1 Option Delta (Δ)

First-order sensitivity of the option price to the underlying price

1.1 What is Delta?

Delta is the change in the option price for a ₹1 change in the price of the underlying asset, holding everything else (time, volatility, rates) constant. Mathematically it is the **first derivative** (slope) of the option-price curve with respect to S.

$$\Delta = \partial V / \partial S \approx \text{(Change in option price) / (Change in underlying price)}$$

Intuition

If a call has Δ = 0.60, then a ₹1 rise in the share price raises the call price by about ₹0.60. Equivalently, **one call behaves like 0.60 shares** — so Delta is also the **hedge ratio**: the number of shares needed to hedge one option.

1.2 Black-Scholes-Merton Delta: N(d1) and N(d1) − 1

$$d_1 = [\ \ln(S/K) + (r + \sigma^2/2)\cdot T\ ] / (\sigma\cdot\sqrt{T})$$

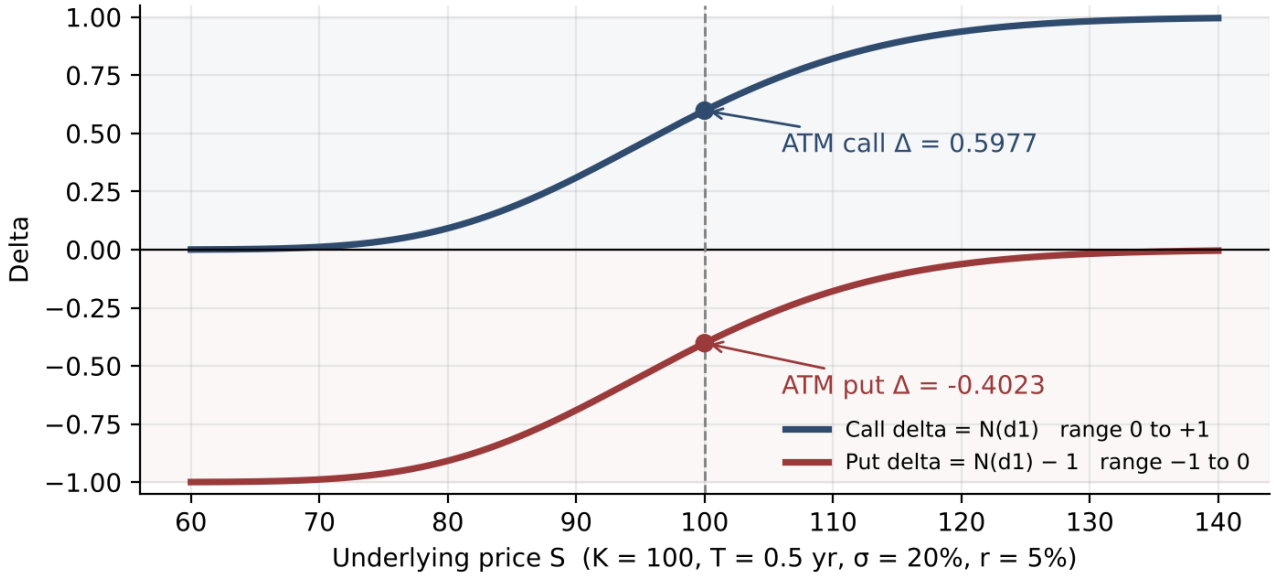
	Call option	Put option
Delta formula	$\Delta_{call} = N(d_1)$	$\Delta_{put} = N(d_1) - 1$
Range	0 to +1	−1 to 0
Deep out-of-the-money	≈ 0	≈ 0
At-the-money	≈ +0.5 (slightly above, due to drift)	≈ −0.5
Deep in-the-money	≈ +1 (moves like the share)	≈ −1 (moves like a short share)
Excel	=NORM.S.DIST(d1, TRUE)	=NORM.S.DIST(d1, TRUE) − 1

Useful check

Because $\Delta_{call} - \Delta_{put} = N(d_1) - [N(d_1) - 1] = \mathbf{1}$, put-call parity ($C - P = S - Ke^{-rT}$) holds for deltas too: a long call + short put has the delta of one share.

Worked example — Option A (S = 100, K = 100, r = 5%, σ = 20%, T = 0.5):

1	ln(S/K) = ln(100/100) = **0**
2	$(r + \sigma^2/2)\cdot T = (0.05 + 0.02) \times 0.5 =$ **0.0350**
3	$\sigma\cdot\sqrt{T} = 0.20 \times \sqrt{0.5} = 0.20 \times 0.70711 =$ **0.14142**
4	$d_1 = (0 + 0.0350) / 0.14142 =$ **0.2475**
5	Call delta = N(0.2475) = **0.5977** Put delta = 0.5977 − 1 = **-0.4023**



1.3 Position Delta — long and short positions

A portfolio's delta is simply the **quantity-weighted sum** of the deltas of its components. A share has Δ = +1, a short share −1. Selling an option **flips the sign** of its delta.

Position Δ = Σ n_i × $Δ_i$ (× lot size) n > 0 long, n < 0 short

Position (1,000 units)	Δ per unit	Position Δ	Sign	Gains when...
Long call A	+0.5977	+597.7	+	S rises (bullish)
Short call A	+0.5977	−597.7	−	S falls (bearish)
Long put A	-0.4023	−402.3	−	S falls (bearish)
Short put A	-0.4023	+402.3	+	S rises (bullish)
Long shares	+1	+1,000.0	+	S rises
Short shares	+1	−1,000.0	−	S falls

Example — reading a position delta

A trader is **long 1,000 calls and long 1,000 puts** (a long straddle) on A. Position Δ = 1,000 × 0.5977 + 1,000 × (-0.4023) = **+195.5**. The book behaves like holding about 195 shares: a ₹1 rise in S gains ≈ ₹195, a ₹1 fall loses ≈ ₹195.

Implications of long vs short positions

Positive position delta → the book profits when the underlying rises (net long the market). **Negative position delta** → profits when it falls (net short). **Zero** → insensitive to small moves: the book is **delta neutral**. Long calls and short puts add + delta; short calls and long puts add − delta.

1.4 Delta-Neutral Portfolio

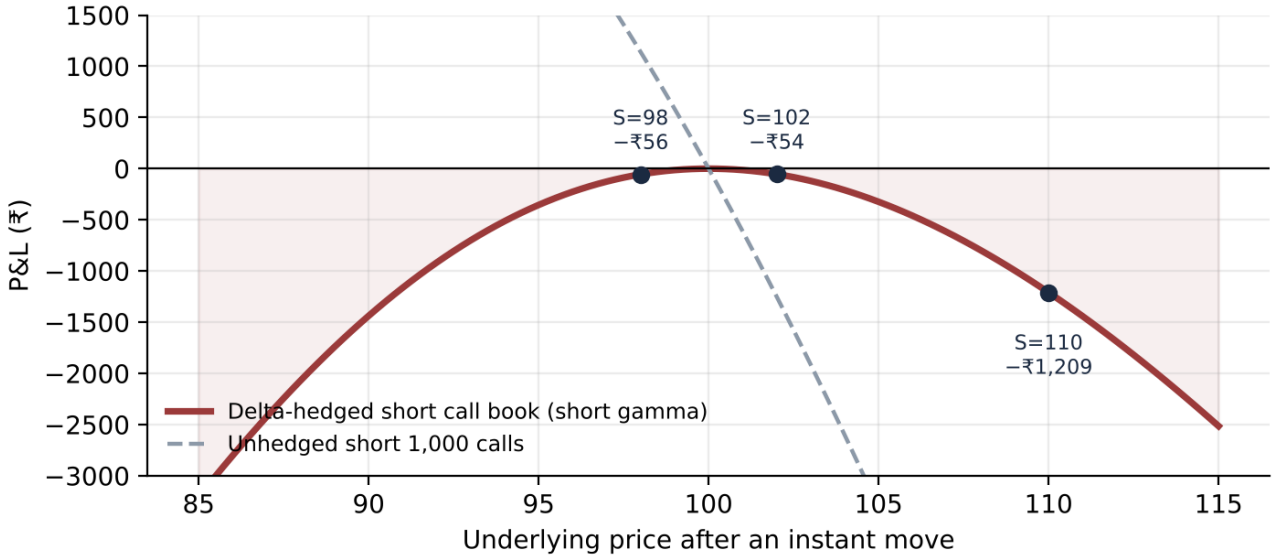
A **delta-neutral** portfolio has a total position delta of zero, so its value does not change for a **small** move in the underlying. The usual way to achieve it is to trade the underlying shares (Δ = 1) against the options.

Shares to trade = − (Position Δ of the options)

1	A bank **writes (sells) 1,000 calls of A** to a client at ₹6.8887 each.
2	Position delta = −1,000 × 0.5977 = **−597.73** (the bank loses if S rises).
3	To neutralise, **buy 597.73 shares** (Excel: `=-(-1000*NORM.S.DIST(d1,TRUE))`).
4	New position delta = −597.73 + 597.73 = **0** ✔

Three illustrations — what happens when the price moves instantly?

	Illustration 1 S: 100 → 102	Illustration 2 S: 100 → 98	Illustration 3 S: 100 → 110
New call price (BSM)	₹8.1378	₹5.7489	₹14.0754
P&L on short 1,000 calls	−₹1,249	₹1,140	−₹7,187
P&L on 597.73 shares	₹1,195	−₹1,195	₹5,977
Net P&L of hedged book	**−₹54**	**−₹56**	**−₹1,209**
New call delta $N(d_1)$	0.6508	0.5417	0.8216
New position delta	-53.08	+56.07	-223.85
Rebalancing action	Buy 53.08 shares	Sell 56.07 shares	Buy 223.85 shares



What the three illustrations teach

Illustrations 1 & 2 (small moves): the share gain almost exactly offsets the option loss (and vice versa) — the hedge works; the residual ≈ ₹54–56 is tiny relative to the ₹1,195 move in each leg.
Illustration 3 (large move): the hedge leaks **−₹1,209**. The call's delta rose from 0.5977 to 0.8216, but our share hedge stayed fixed. The option price is a **curve**, the share hedge is a **straight line**.
Lesson: delta neutrality holds only for small moves and only **instantaneously**. It must be **rebalanced** (dynamic hedging) — and note the short-option book loses whichever way S moves. That curvature is **Gamma** → Section 2.

2 Option Gamma (Γ)

Second-order sensitivity — the rate of change of Delta

2.1 What is Gamma? Why a 2nd-order derivative?

Gamma measures how fast Delta changes when the underlying price changes. Since Delta is already the first derivative of the option price, Gamma is the **second derivative** of the option price with respect to S — it measures the **curvature** (convexity) of the option-price curve.

$$\Gamma = \partial\Delta / \partial S = \partial^2 V / \partial S^2$$

Analogy

Think of a car: **Delta is the speed** (₹ of option value per ₹ of share move) and **Gamma is the acceleration** (how quickly that speed changes). High gamma = delta changes fast = the hedge goes stale fast.

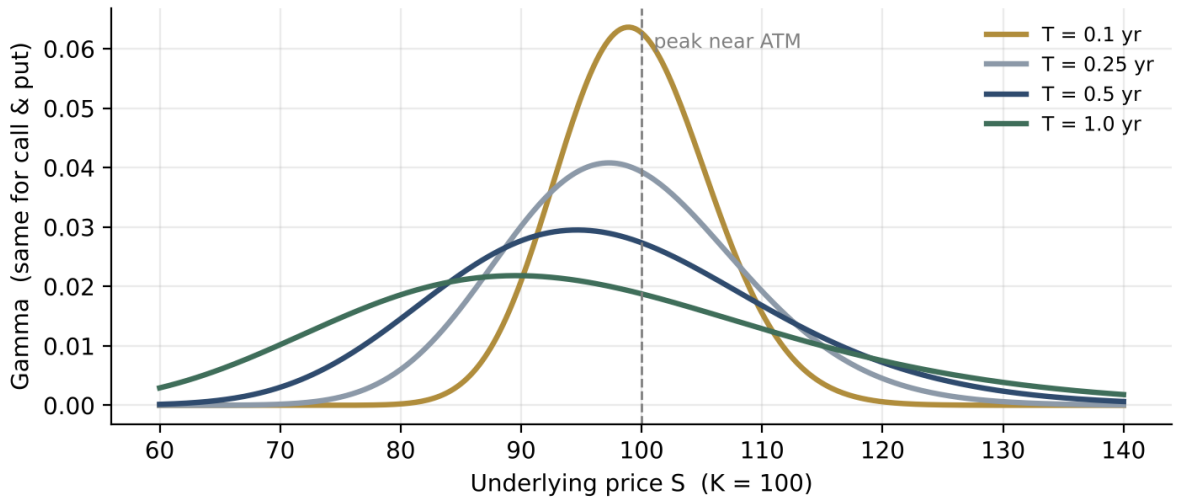
2.2 Gamma for a Call and a Put

$$\Gamma_{call} = \Gamma_{put} = N'(d_1) / (S \cdot \sigma \cdot \sqrt{T}) \text{ where } N'(x) = e^{-x^2/2} / \sqrt{(2\pi)}$$

Gamma is **identical for a call and a put** with the same K and T: $\Delta_{put} = \Delta_{call} - 1$, and differentiating a constant (−1) gives zero. Gamma is always **positive for a long option** and is largest for **at-the-money, short-dated** options.

Worked example — Option A:

1	d_1 = 0.2475 (from Section 1)
2	$N'(d_1) = e^{-(0.2475)^2/2} / \sqrt{(2\pi)}$ = **0.38691** Excel: `=NORM.S.DIST(d1,FALSE)`
3	$S \cdot \sigma \cdot \sqrt{T}$ = 100 × 0.20 × 0.70711 = **14.1421**
4	Γ = 0.38691 / 14.1421 = **0.02736**
5	Meaning: if S rises ₹1 → Δ rises from 0.5977 to ≈ 0.6251 (exact BSM: 0.6247)



2.3 Sign of Gamma — impact on the position

Position	Gamma sign	Effect of a big move (either direction)	Pays / earns time decay
Long call / Long put	**+ (long gamma)**	Gains — delta moves in your favour	Pays theta
Short call / Short put	**− (short gamma)**	Loses — delta moves against you	Earns theta
Shares / Futures	**0**	Linear; delta fixed at ±1	—
Our book: short 1,000 A	**−27.36**	Loses on large moves (Section 1.4!)	Earns theta

2.4 Delta + Gamma together: the Taylor approximation

$$\Delta V \approx \Delta \cdot (dS) + \tfrac{1}{2} \cdot \Gamma \cdot (dS)^2$$

S: 100 → 105 (dS = 5)	Calculation	Value
Delta term	0.5977 × 5	₹2.9887
Gamma term	½ × 0.02736 × 25	₹0.3420
Delta + Gamma estimate		₹3.3307
Exact BSM change	C(105) − C(100)	**₹3.3125**

Why the delta-neutral book lost money

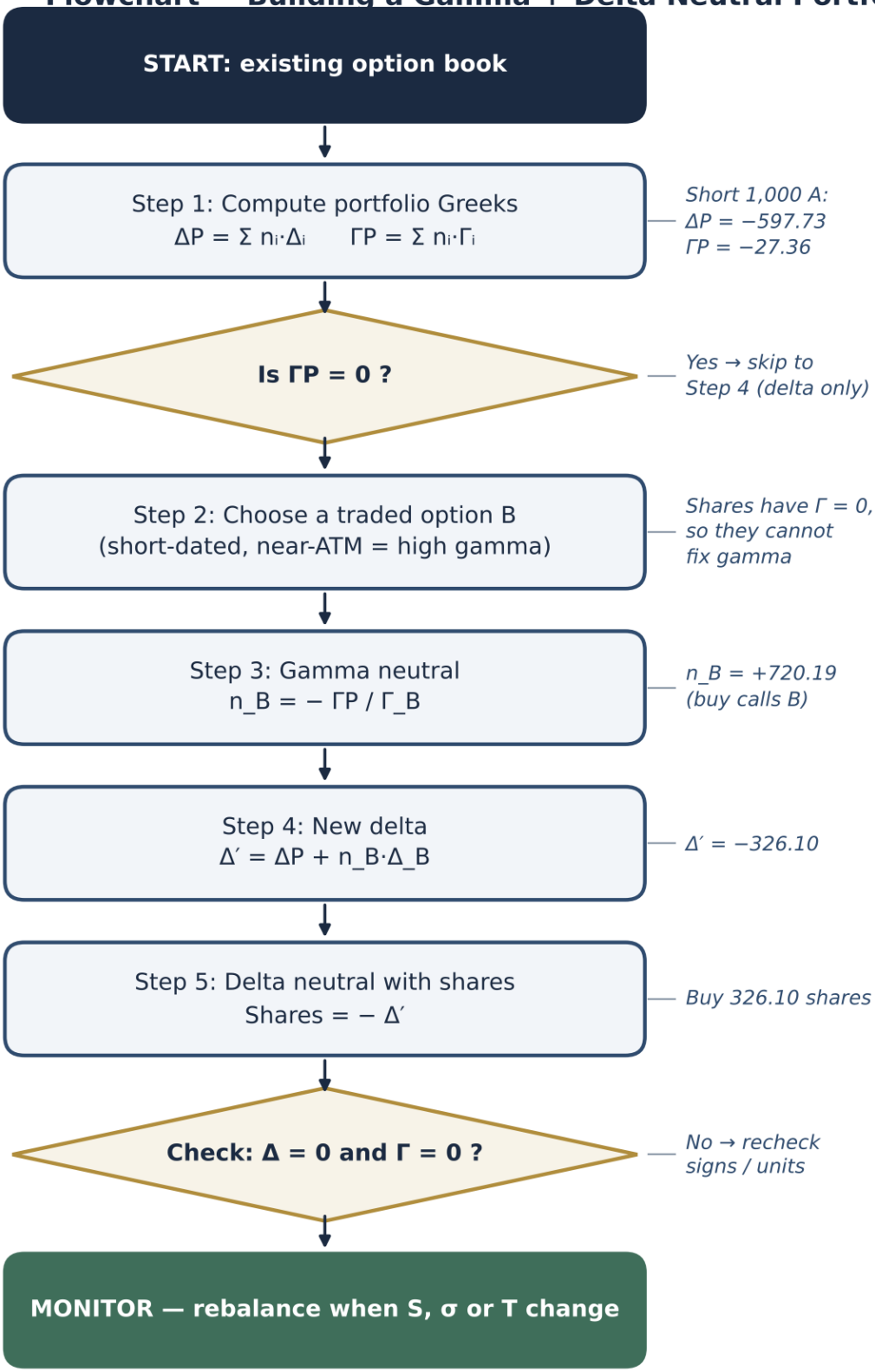
Delta alone (₹2.99) under-states the true change (₹3.31). Adding the gamma term recovers almost all of it. For a delta-neutral book the first term is zero, so **P&L ≈ ½·Γ·(dS)²** — negative for every move when Γ < 0. Check Illustration 3: ½ × (−27.36) × 10^2 = −₹1,368 vs actual −₹1,209.

2.5 Building a Gamma + Delta Neutral Portfolio

Golden rule — order matters: Gamma first (options), Delta last (shares)

Shares have **zero gamma**, so they cannot fix gamma. Gamma must be neutralised **first** using another **option**; that option adds delta, which is then neutralised with **shares** (which do not disturb gamma).

Flowchart — Building a Gamma + Delta Neutral Portfolio

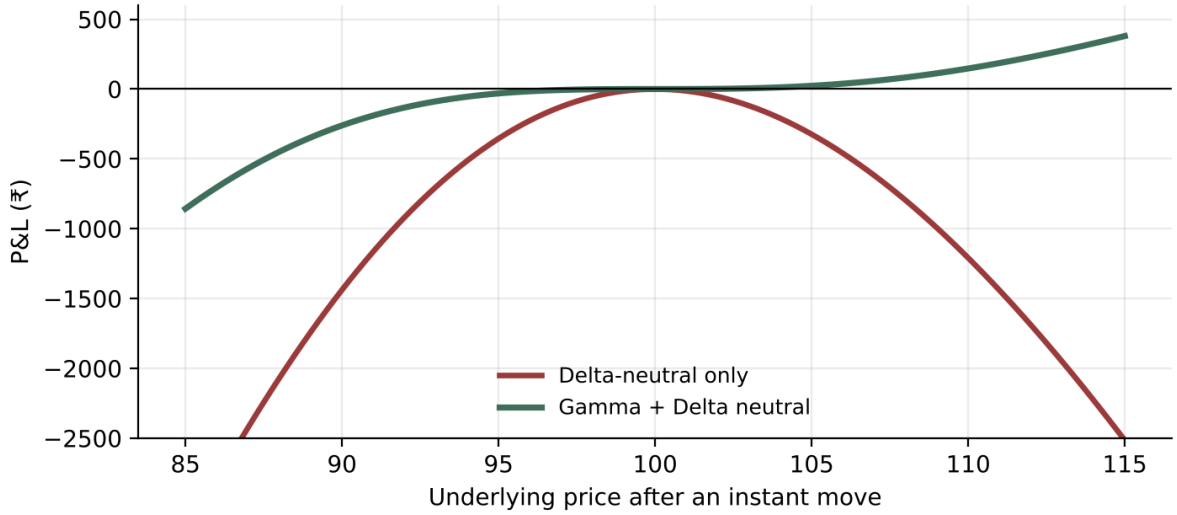


$$n_B = -\Gamma_{Portfolio} / \Gamma_B$$
$$\text{then shares} = -(\Delta_{Portfolio} + n_B \cdot \Delta_B)$$

1	Existing book: short 1,000 calls A → $\Delta_P = -597.73$, $\Gamma_P = -27.3587$
2	Hedge option B (K=105, T=0.25): $\Delta_B = 0.3772$, $\Gamma_B = 0.03799$
3	Gamma neutral: $n_B = -(-27.3587) / 0.03799$ = **buy 720.19 calls B**
4	New delta = −597.73 + 720.19 × 0.3772 = **−326.10**
5	Delta neutral: **buy 326.10 shares** → Δ = 0 and Γ = 0 ✔

```
Excel (see sheet 4_GammaDeltaNeutral):
Portfolio gamma     =-1000*Gamma_A
n_B (buy calls B)   =-PortfolioGamma/Gamma_B
Delta after n_B     =-1000*Delta_A + n_B*Delta_B
Shares to buy       =-DeltaAfter_nB
Check gamma         =-1000*Gamma_A + n_B*Gamma_B + Shares*0   → 0
```

Instant move in S	Delta-neutral only	Gamma + Delta neutral
S → 90	−₹1,438	**−₹260**
S → 95	−₹354	**−₹31**
S → 98	−₹56	**−₹2**
S → 102	−₹54	**₹2**
S → 105	−₹324	**₹23**
S → 110	−₹1,209	**₹147**



Result

With only a delta hedge the book lost −₹1,438 at S = 90 and −₹1,209 at S = 110. Adding the gamma hedge shrinks those to −₹260 and ₹147: the book now stays neutral over a **much wider range** and needs rebalancing far less often.

3 Option Vega (ν)

Sensitivity of the option price to volatility

3.1 What is Vega?

Vega is the change in the option price for a change in the volatility of the underlying. (Vega is not a real Greek letter; some texts call it *kappa*.) Higher volatility makes both calls and puts more valuable, so **Vega is positive for long calls and long puts** and the **same** for both.

$$\textbf{Vega} = \partial V / \partial \sigma = S \cdot \sqrt{T} \cdot N'(d_1)$$

Units — read this carefully

Vega from the formula is per **1.00** (i.e. 100 percentage points) of volatility. Divide by 100 for the practitioner's "vega per 1 vol point" (σ moving from 20% to 21%).

3.2 How to calculate Vega — two ways

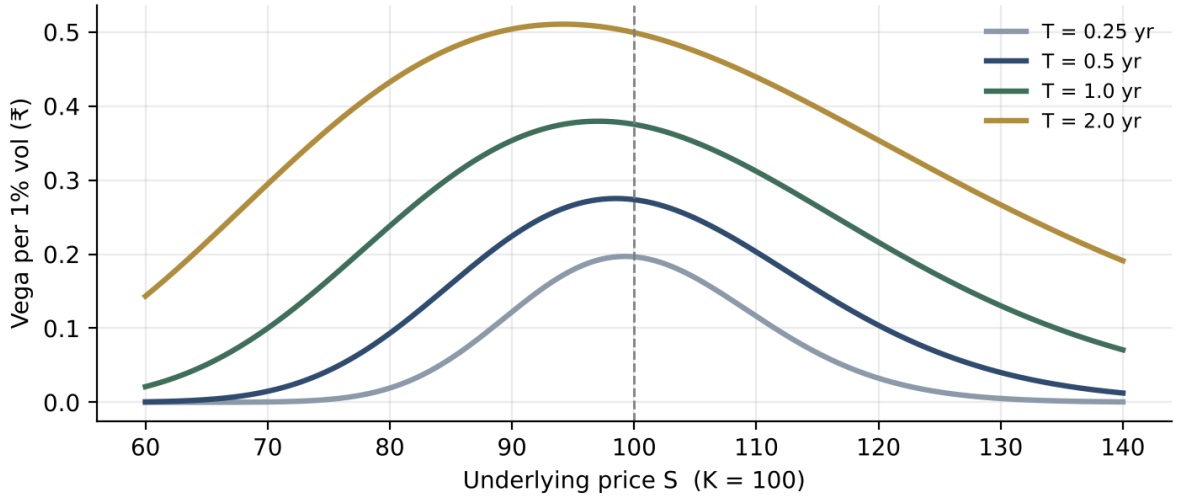
Method 1 — Formula (Option A)

1	$S \cdot \sqrt{T} = 100 \times 0.70711 = 70.711$
2	$N'(d_1) = 0.38691$ (same value used for Gamma)
3	Vega = 70.711 × 0.38691 = **27.3587** per 1.00 of σ → **₹0.2736** per 1 vol point
4	Position vega of short 1,000 A = −1,000 × 0.2736 = **−₹273.59** per vol point

Method 2 — Re-price ("bump") the option

Volatility	BSM call price	Change
σ = 20%	₹6.8887	
σ = 21%	₹7.1625	**₹0.2738** (≈ formula ₹0.2736) ✔

```
Excel:  Vega (per 1.00) =S*SQRT(T)*NORM.S.DIST(d1,FALSE)
        Vega (per 1%)    =S*SQRT(T)*NORM.S.DIST(d1,FALSE)/100
```



Key contrast — Gamma vs Vega

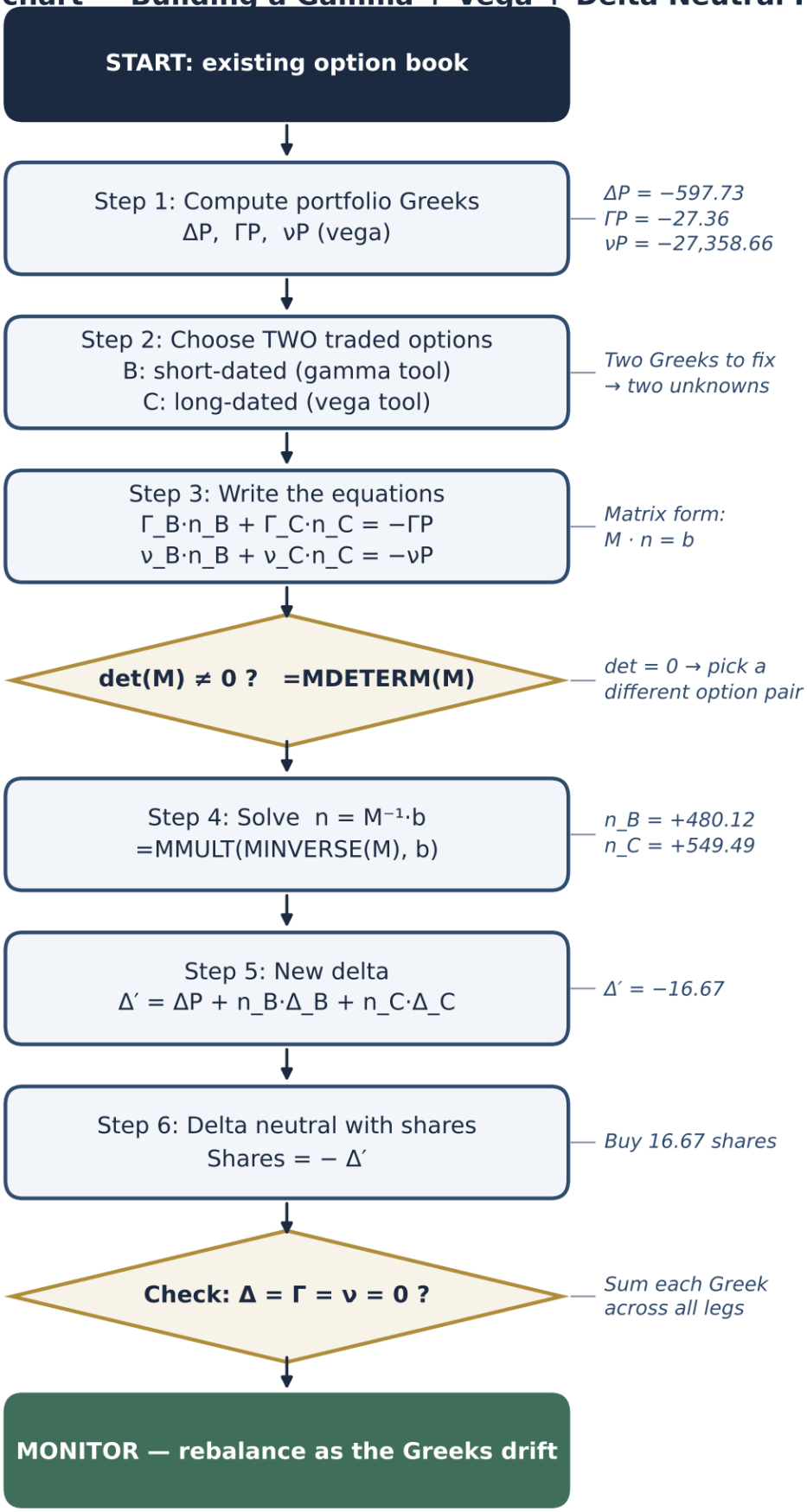
Gamma is largest for **short-dated** ATM options; **Vega** is largest for **long-dated** ATM options. So a short-dated option is a "gamma tool" and a long-dated option is a "vega tool". This is exactly why we need **two different options** to neutralise Gamma and Vega at the same time.

3.3 Vega neutral alone is not enough

Using only option C: n_C = 1,000 × 27.359 / 33.193 = 824.24. Vega becomes zero, but Gamma = −27.359 + 824.24 × 0.01660 = **-13.679** — still short gamma. One option can fix one Greek. **Two Greeks (Γ and ν) need two options**; delta is then fixed with shares.

3.4 Gamma + Vega + Delta Neutral — solving the equations

Flowchart — Building a Gamma + Vega + Delta Neutral Portfolio



We hold short 1,000 A. Buy n_B of option B and n_C of option C so that the total gamma and vega are zero:

Greek (per option)	Option A	Option B (K=105, 3m)	Option C (K=95, 12m)
Delta $N(d_1)$	0.5977	0.3772	0.7279
Gamma	0.02736	0.03799	0.01660
Vega (per 1.00)	27.3587	18.9941	33.1927

Gamma: $0.03799 \cdot n_B + 0.01660 \cdot n_C = 1{,}000 \times 0.02736 = 27.3587$

Vega: $18.9941 \cdot n_B + 33.1927 \cdot n_C = 1{,}000 \times 27.3587 = 27{,}358.66$

Step-by-step solution by hand (Cramer's rule)

1	Determinant = $\Gamma_B \cdot \nu_C - \Gamma_C \cdot \nu_B = 1.2609 - 0.3152 =$ **0.9457**
2	$n_B = (b_1 \cdot \nu_C - \Gamma_C \cdot b_2) / \text{det} = (908.109 - 454.054) / 0.9457 =$ **480.12** (buy)
3	$n_C = (\Gamma_B \cdot b_2 - \nu_B \cdot b_1) / \text{det} = (1039.309 - 519.654) / 0.9457 =$ **549.49** (buy)
4	Delta after options = $-597.73 + 480.12 \times 0.3772 + 549.49 \times 0.7279 =$ **-16.67**
5	**Buy 16.67 shares** → $\Delta = 0$, $\Gamma = 0$, Vega = 0 ✔

The same solution in matrix form with Excel MINVERSE and MMULT

$$[\, n_B \,;\, n_C \,] = M^{-1} \cdot b$$

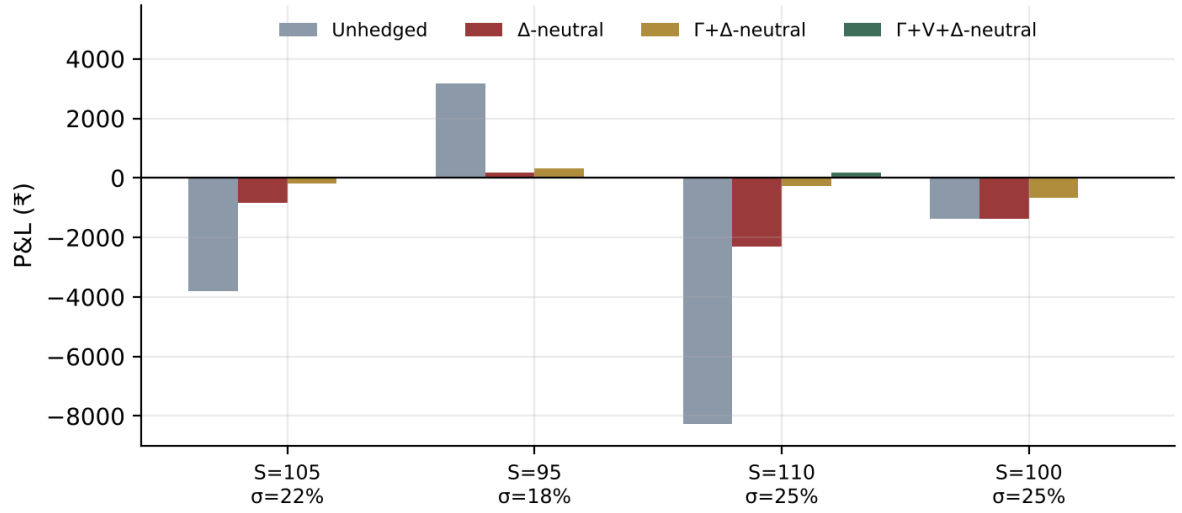
$$M = [\, \Gamma_B \; \Gamma_C \,;\, \nu_B \; \nu_C \,],\; b = [\, 1000 \cdot \Gamma_A \,;\, 1000 \cdot \nu_A \,]$$

```
1. Put M in a 2x2 block, e.g. C24:D25 (Gamma row, Vega row) and b in F24:F25
2. Inverse: select 2x2 block -> =MINVERSE(C24:D25)   (Ctrl+Shift+Enter in older Excel)
     M^-1 = [ 35.0985   -0.017549 ;  -20.0847   0.040169 ]
3. Quantities: select 2x1 block -> =MMULT(MINVERSE(C24:D25),F24:F25)
     n_B = 480.12     n_C = 549.49
4. Determinant check: =MDETERM(C24:D25)  (non-zero => unique solution)
5. Shares: =-( -1000*Delta_A + n_B*Delta_B + n_C*Delta_C )
```

Leg	Quantity	Delta	Gamma	Vega (per 1.00)
Short calls A	−1,000.00	-597.73	-27.3587	-27,358.66
Long calls B	+480.12	181.09	18.2391	9,119.55
Long calls C	+549.49	399.97	9.1196	18,239.11
Long shares	+16.67	16.67	0.0000	0.00
TOTAL		**0.00**	**0.0000**	**0.00**

Stress test — price and volatility shocked together (instantaneous)

Scenario	Unhedged	Δ-neutral	Γ+Δ-neutral	Γ+ν+Δ-neutral
S → 105, σ → 22%	−₹3,816	−₹827	−₹183	**₹38**
S → 95, σ → 18%	₹3,166	₹177	₹314	**−₹1**
S → 110, σ → 25%	−₹8,278	−₹2,300	−₹277	**₹182**
S → 100, σ → 25%	−₹1,371	−₹1,371	−₹678	**₹25**



What the stress test shows

Each additional neutrality removes one source of risk. Only the **Gamma + Vega + Delta neutral** book is protected against a joint move in price **and** volatility (e.g. a sell-off with a volatility spike). Note the last scenario: with S unchanged and σ up 5 points, the Δ-neutral book still loses ₹1,371 — pure vega risk.

4 Why not Theta-neutral or Rho-neutral?

Theta (Θ = ∂V/∂t) and Rho (ρ = ∂V/∂r)

4.1 Theta — time decay is not a risk

$$\Theta_{call} = -S \cdot N'(d_1) \cdot \sigma / (2\sqrt{T}) - r \cdot K \cdot e^{-rT} \cdot N(d_2)$$

For option A, Θ = −5.4717 − 2.6442 = **-8.1160** per year = **₹-0.0222** per day. Our short 1,000 calls therefore **earn** about ₹22.24 per day from time decay.

1	**Time is certain, not random.** Tomorrow will arrive with probability 1. Risk management hedges **uncertain** variables (S, σ, r). The theta P&L is known in advance — there is nothing to hedge, and no instrument lets you "sell" the passage of time.
2	**Theta is tied to Gamma by the Black-Scholes PDE:** Θ + r·S·Δ + ½·σ²·S²·Γ = r·V. For a delta-neutral book Θ = r·Π − ½·σ²·S²·Γ. So once Γ is managed, Θ is **determined automatically** — it is not an independent risk. Short gamma ⇔ positive theta; long gamma ⇔ negative theta (theta is the "rent" for gamma).
3	**Neutralising theta would destroy the other hedges:** it needs yet another option, which re-introduces gamma and vega exposure, for no reduction in risk.

Check the PDE for option A	Value
Θ (per year)	-8.1160
+ r·S·Δ = 0.05 × 100 × 0.5977	+2.9887
+ ½·σ²·S²·Γ = 0.5 × 0.04 × 10,000 × 0.02736	+5.4717
= Left-hand side	**0.3444**
Right-hand side r·V = 0.05 × 6.8887	**0.3444** ✔

4.2 Rho — a small, slow-moving risk managed elsewhere

$$\rho_{call} = K \cdot T \cdot e^{-rT} \cdot N(d_2)$$
$$\rho_{put} = -K \cdot T \cdot e^{-rT} \cdot N(-d_2)$$

For option A, ρ = 100 × 0.5 × $e^{-0.025}$ × 0.5422 = 26.4424 per 1.00 → **₹0.2644** per 1% change in rates (similar in size to vega of ₹0.2736 per vol point).

1	**Rates move far less than volatility.** A 0.25% policy-rate change vs a 5-point volatility jump: rate impact ≈ ₹66 on 1,000 options, volatility impact ≈ ₹1,368.
2	**Rho is small for short-dated equity/index options** (it scales with T). It matters mainly for long-dated options and for interest-rate derivatives — where it is managed as part of the rate book.
3	**Better hedge instruments exist:** interest-rate risk is hedged at the desk / ALM level with bonds, futures, FRAs and swaps (DV01/PV01 framework), not by trading more options.
4	**Cost vs benefit:** each extra option leg adds transaction cost, bid-ask spread and model risk. Desks therefore **monitor** Theta and Rho and **hedge** Delta, Gamma and Vega.

4.3 Practitioner summary

Greek	Measures	Long call / Long put	Hedge with	How often
Delta Δ	∂V/∂S	+ / −	Underlying shares, futures	Daily or intraday
Gamma Γ	∂²V/∂S²	+ / +	Short-dated options	Periodically
Vega ν	∂V/∂σ	+ / +	Long-dated options	Periodically
Theta Θ	∂V/∂t	− / − (usually)	Not hedged — it is deterministic	Monitored
Rho ρ	∂V/∂r	+ / −	Bonds, swaps, IR futures (rate book)	Monitored

4.4 Excel functions cheat-sheet

Purpose	Excel formula
d1	=(LN(S/K)+(r+sigma^2/2)*T)/(sigma*SQRT(T))
d2	=d1-sigma*SQRT(T)
N(d1) — call delta	=NORM.S.DIST(d1,TRUE)
N(d1) − 1 — put delta	=NORM.S.DIST(d1,TRUE)-1
N′(d1) — standard normal density	=NORM.S.DIST(d1,FALSE)
Gamma	=NORM.S.DIST(d1,FALSE)/(S*sigma*SQRT(T))
Vega (per 1%)	=S*SQRT(T)*NORM.S.DIST(d1,FALSE)/100
Call price	=S*NORM.S.DIST(d1,TRUE)-K*EXP(-r*T)*NORM.S.DIST(d2,TRUE)
Solve hedge quantities	=MMULT(MINVERSE(M),b) and =MDETERM(M)

Companion Excel workbook

Every number in this handout is reproduced with live formulas in the companion workbook **Option_Greeks_Basics.xlsx**. Change S, σ, r, K or T on the **Inputs_BS** sheet and all Greeks, hedges and illustrations update automatically.

5 Three Simple Caselets — with solutions

Apply Delta, Gamma and Vega hedging step by step (solutions in sheet 8_Caselets)

Caselet 1: The put writer at a brokerage desk

The situation

A dealing desk has **written (sold) 2,000 puts** on a stock trading at ₹100 (strike ₹100, 6 months, σ = 20%, r = 5%), receiving ₹4.4197 per put. The risk manager wants the book delta-neutral. The next morning the stock opens at **₹97**.

Required

(a) Position delta of the short puts. (b) Hedge trade in shares. (c) P&L of the hedged book at ₹97. (d) Rebalancing trade.

Solution

1	Put delta = $N(d_1) - 1 = 0.5977 - 1 =$ **-0.4023**
2	Position delta = −2,000 × (-0.4023) = **+804.53** (short puts are bullish)
3	Hedge: **short-sell 804.53 shares** so that total delta = 0
4	At S = 97: put price = ₹5.7526 → P&L on puts = −2,000 × (5.7526 − 4.4197) = −₹2,666
5	P&L on short shares = −804.53 × (97 − 100) = ₹2,414 → **net −₹252**
6	New put delta = -0.4872 → position delta = −2,000 × (-0.4872) − 804.53 = **+169.85**

Answer

(a) +804.53 (b) Short 804.53 shares (c) Net −₹252 — the hedge absorbed most of the −₹2,666 option loss; the remainder is the cost of short gamma (d) **Short a further 169.85 shares**.

Caselet 2: Making a book Gamma + Delta neutral

The situation

A trader's option book has **Δ = +400** and **Γ = -250**. A traded option X is available with **Δ = 0.55** and **Γ = 0.025**. Shares of the underlying can also be traded.

Required

How many options X and how many shares must be traded to make the book both gamma-neutral and delta-neutral?

Solution

1	Gamma first (shares have no gamma): $n_X = -\Gamma_P / \Gamma_X = -(-250) / 0.025 =$ **+10,000** → buy 10,000 options
2	New delta = 400 + 10,000 × 0.55 = **5,900**
3	Delta last: shares = −5,900 → **sell 5,900 shares**
4	Check: Γ = -250 + 10,000 × 0.025 = 0; Δ = 5,900 − 5,900 = 0 ✔

Answer

Buy 10,000 options X and sell 5,900 shares. Notice how buying options for gamma created a large delta that the shares then had to remove — the order of steps matters.

Caselet 3: Gamma + Vega + Delta neutral with two options

The situation

A portfolio has **Δ = +200**, **Γ = -600** and **Vega = -3,000** (vega per 1% vol). Two traded options: **X** (short-dated) Δ = 0.5, Γ = 0.04, Vega = 0.1; **Y** (long-dated) Δ = 0.6, Γ = 0.02, Vega = 0.25.

Required

Find the positions in X, Y and the underlying that make the portfolio gamma, vega and delta neutral.

Solution

1	Equations: 0.04·x + 0.02·y = 600 (gamma) and 0.1·x + 0.25·y = 3,000 (vega)
2	Determinant = 0.04×0.25 − 0.02×0.1 = **0.008** (≠ 0, unique solution)
3	x = (600×0.25 − 0.02×3,000) / 0.008 = **11,250**; y = (0.04×3,000 − 0.1×600) / 0.008 = **7,500**
4	Excel: `=MMULT(MINVERSE({0.04,0.02;0.10,0.25}),{600;3000})` → 11,250 ; 7,500
5	New delta = 200 + 11,250×0.5 + 7,500×0.6 = **10,325** → **sell 10,325 shares**
6	Check: Γ = −600 + 450 + 150 = 0; Vega = −3,000 + 1,125 + 1,875 = 0; Δ = 0 ✔

Answer

Buy 11,250 of X, buy 7,500 of Y, sell 10,325 shares. X (high gamma per unit of vega) does most of the gamma work; Y (high vega) does most of the vega work.

Try it yourself

For each caselet, change one input (e.g. the stock opens at ₹103 in Caselet 1, or $\Gamma_P = -800$ in Caselet 3) in sheet **8_Caselets** and explain how and why the hedge changes.